MONTHLY WEATHER REVIEW

ALFRED J. HENRY, Editor.

VOL. 51, No. 9. W. B. No. 818. | SEPTEMBER, 1923. | CLOSED NOV. 3, 1923. ISSUED DEC. 5, 1923.

THE LAW OF PRESSURE RATIOS AND ITS APPLICATION TO THE CHARTING OF ISOBARS IN THE LOWER LEVELS OF THE TROPOSPHERE.

C. LEROY MEISINGER, Meteorologist.

[Weather Bureau, Washington, D. C., August 3, 1923.]

INTRODUCTION.

The solution of many problems of theoretical and practical meteorology is hindered by a lack of information concerning the simultaneous vertical and geographical distribution of certain meteorological elements. One of these elements, barometric pressure, has always been regarded as of supreme importance in weather forecasting; and, with the increasing availability of aerological data, there has developed a belief that barometric conditions at certain free-air levels may possess prognostic value with respect to surface weather. The elevation of these strategic levels is not definitely known, but the importance of the problem and the significance of the possible results abundantly justify an attempt to ascertain the nature of horizontal barometric distributions at the greatest possible heights in the atmosphere.

It is a relatively easy matter to secure information concerning the vertical distribution of meteorological elements at a small number of aerological stations; but it is difficult to produce daily charts of the geographical distribution of these elements. Recently published papers have attempted to achieve this result for the levels 1 and 2 kilometers (3,281 and 6,562 feet) above sea level,[1] and subsequent study of the accuracy of the maps thus produced indicates that they are reliable.[2] The attainment of this objective, however, serves only as a stimulus to the accomplishment of similar results for higher levels. Yet, one encounters difficulties owing to paucity of data, in attempting to extend the original method to levels higher than 2 kilometers above sea level. It has seemed that a more fruitful field might be found in dealing directly with pressures than with the estimation of air-column temperatures to such great heights. In other words, previous studies have provided a means of securing a knowledge of the barometric distribution at 1 and 2 kilometers above sea level. Is it possible, with the pressure at these two levels, in addition to the precisely-measured surface pressure, to learn something about the pressure distribution at a fourth, and higher, level? The present paper seeks an answer to this question.

PRELIMINARY CONSIDERATIONS.

Ratios between pressures at different free-air levels.—In order to secure a foothold for an attack upon this problem, mean monthly free-air pressures for various levels and for the several aerological stations of the Weather Bureau were examined. It was found that if a ratio is formed between the pressure at some high level, p_z, and the surface, p_s, and between the pressure at 2 kilometers above sea level, p_2, and 1 kilometer above sea level, p_1,

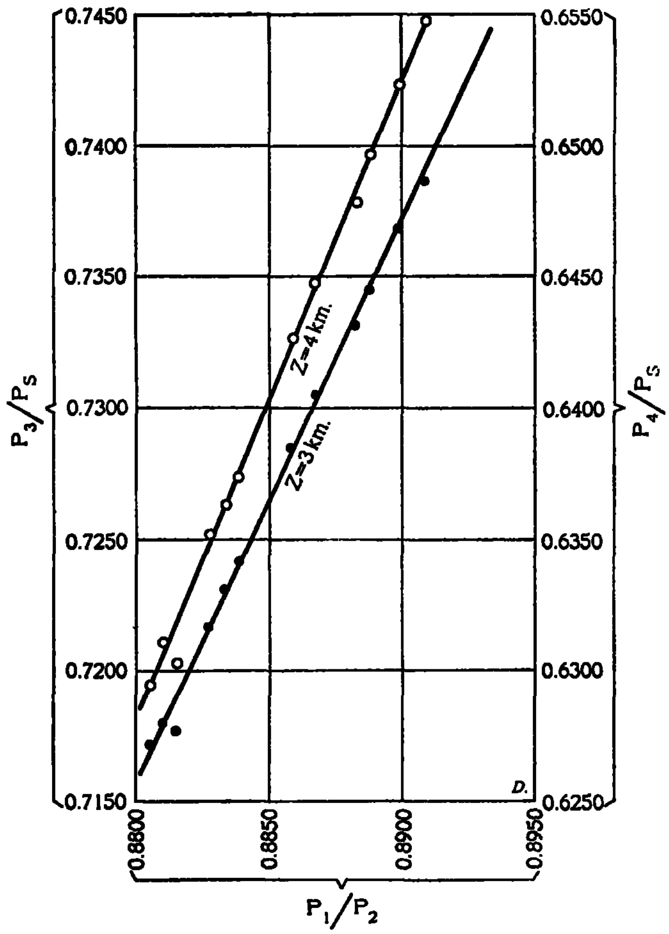

FIG. 1.—The relation between p_z/p_s and p_2/p_1 (annual means) at Drexel, Nebr.

the ratio p_z/p_s stands in an apparently linear relation to p_2/p_1. In other words,

$$p_z/p_s = (ap_2/p_1) + b,$$

in which a and b are constants. Thus, if p_s, p_1, p_2, a, and b were known, p_z could readily be computed from the equation

$$p_z = p_s[(ap_2/p_1) + b] \qquad (1)$$

Should such a simple relation hold for pressures at any time, as well as for monthly means, it is seen at once that

[1] Meisinger, C. LeRoy: The preparation and significance of free-air pressure maps for the central and eastern United States. MO. WEATHER REV. SUPPLEMENT NO. 21, Washington, 1922.

[2] *Ibid:* Concerning the accuracy of free-air pressure maps. MO. WEATHER REV., April, 1923, pp. 190–199.

70169—23——1

here is a method of great promise for computing free-air pressures at high levels upon the basis of surface conditions only.[3]

Figure 1 shows for the Drexel (Nebraska) aerological station the relation between these ratios when z is respectively 3 and 4 kilometers (9,842 and 13,123 feet) above sea level. For these two levels the points lie very close to the line of best fit, but, in considering a similar relation for the 5-kilometer (16,404-foot) level,

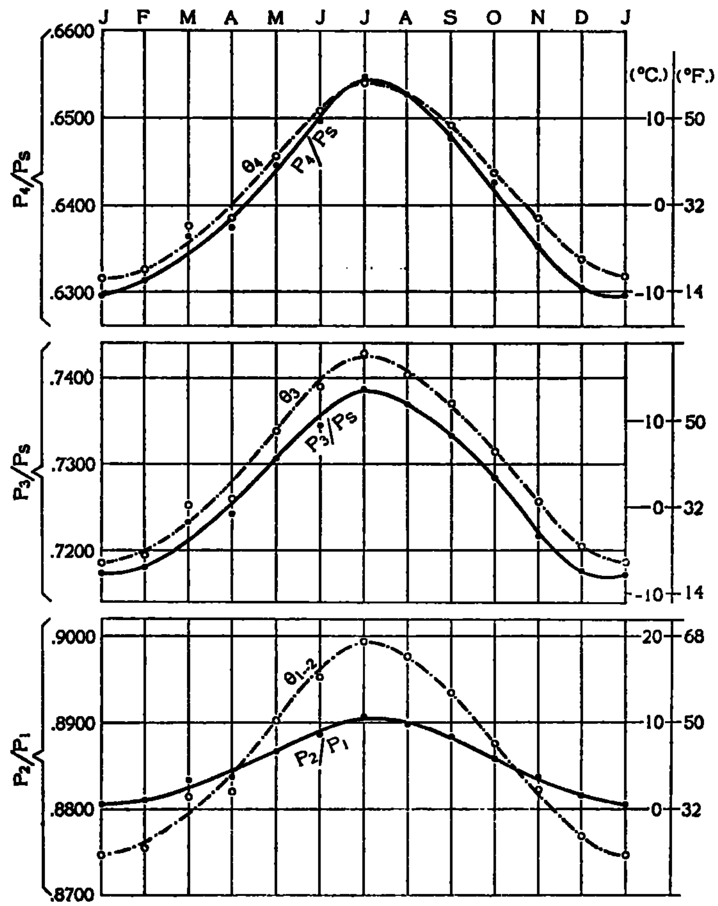

FIG. 2.—Comparison of the annual march of pressures ratios and mean temperatures of the air column at Drexel, Nebr.

the scatter of the dots is slightly greater, owing, probably, to the smaller number of observations upon which means for this level are based.[4]

Annual variation of pressure ratios.—The barometric pressure at any level, considered statically, is the weight of the air above that level. This weight is a function of the temperature of the air between the earth's surface and the level in question, and temperature, whether at some given level in the atmosphere or the average of an air column of stated length, is subject to an annual variation. It is not surprising, therefore, that the annual march of pressure ratios, at least at levels within the range of our interest, is represented by a curve which is quite similar to that of the annual march of mean air-column temperature.

Figure 2 shows the annual march of the several pressure ratios shown in Figure 1, together with the annual march of the mean temperature of the air column pertinent to the particular pressure ratios. The intimate relation that exists between the several curves illustrates clearly the functional relation between temperature and free-air pressure. It will be remembered that in the central and eastern United States the annual march of barometric pressure at the surface is characterized by a minimum in late spring or early summer and a maximum in early winter, characteristics that do not appear either in the annual march of temperature or in the annual march of pressure ratios.

Having considered the monthly mean pressures thus briefly, one's curiosity is whetted by the rather astonishing fact that data which are ordinarily so accurately represented by the exponential law should now appear in linear guise. What is the true nature of this curve which expresses the relation between p_z/p_s and p_2/p_1? The answer to this question can be most readily obtained from theory.

THEORETICAL RELATIONS.

The slope of the curve $y=f(x)$.—The hypsometric relation, the fundamental law in all considerations of this character, may be stated as follows:

$$p_z = p_s \exp \frac{-Z_z}{K\,(1+\alpha\theta_z)} \quad \text{------------} \quad (2)$$

in which p_z and p_s are the barometric pressures at some level in the atmosphere and at the surface, respectively; Z_z is the vertical distance between the point where p_s is measured and the level for which p_z is to be determined, generally known as the length of the air column; K is a constant; α is the coefficient of gas expansion; and θ_z is the virtual mean temperature of the air column.

Since we are interested in the ratio of pressure at some high level to the pressure at the surface, let

$$y = p_z/p_s.$$

Upon substituting the expression for p_z contained in (2) it is found that

$$y = \exp \frac{-Z_z}{K\,(1+\alpha\theta_z)}.$$

While, in ordinary practice, one is interested in some particular level, in which case Z_z would be a parameter, it is best here to consider both θ_z and Z_z as variables. Hence, differentiating, one obtains:

$$dy = \left(\exp \frac{-Z_z}{K(1+\alpha\theta_z)}\right) \frac{Z_z\alpha d\theta_z - (1+\alpha\theta_z)dZ_z}{K(1+\alpha\theta_z)^2} \quad \text{----} \quad (3)$$

Again, let p_m and p_n represent the barometric pressure at two other levels in the atmosphere, and let

$$x = p_m/p_n$$

From (2), as before, it is known that,

$$p_m = p_n \exp \frac{-Z_{m-n}}{K(1+\alpha\theta_{m-n})}$$

whence,

$$x = \exp \frac{-Z_{m-n}}{K(1+\alpha\theta_{m-n})}.$$

Since it is desired to allow the stratum of atmosphere between the bounding surfaces of which Z_{m-n} lies to

[3] It will be recalled that the method for obtaining pressures at 1 and 2 kilometers above sea level makes use only of current pressure, temperature, and wind direction *at the surface*.

[4] All averages of free-air conditions used in this paper have been taken from "An aerological survey of the United States: Part I. Results of observations by means of kites," by W. R. Gregg. MO. WEATHER REV. SUPPLEMENT No. 20, Washington, 1922.

remain fixed regardless of the variations of Z_z, Z_{m-n} may be properly regarded as a parameter, and differentiation will yield the equation,

$$dx = \frac{Z_{m-n}\alpha}{K(1+\alpha\theta_{m-n})^2} \exp \frac{-Z_{m-n}}{K(1+\alpha\theta_{m-n})} \, d\theta_{m-n} \cdots\cdots \quad (4)$$

Dividing (3) by (4) one obtains the slope of the curve representing the relation between y and x:

$$\frac{dy}{dx} = \left(\exp \frac{Z_{m-n}(1+\alpha\theta_z) - Z_z(1+\alpha\theta_{m-n})}{K(1+\alpha\theta_z)\,(1+\alpha\theta_{m-n})}\right) \times \frac{(1+\alpha\theta_{m-n})^2}{(1+\alpha\theta_z)^2} \times \frac{Z_z d\theta_z - (1+\alpha\theta_z dZ_z)}{Z_{m-n} d\theta_{m-n}} \cdots\cdots \quad (5)$$

In case it is desired to treat Z_z as a parameter, the equation (5) is simplified, (since $dZ_z=0$) to

$$\frac{dy}{dx} = \left(\exp \frac{Z_{m-n}(1+\alpha\theta_z) - Z_z(1+\alpha\theta_{m-n})}{K(1+\alpha\theta_z)(1+\alpha\theta_{m-n})}\right) \times \frac{Z_z(1+\alpha\theta_{m-n})^2}{Z_{m-n}(1+\alpha\theta_z)^2} \times \frac{d\theta_z}{d\theta_{m-n}} \cdots\cdots \quad (5a)$$

It is clear from equations (5) and (5a) that the curve representative of the relation between x and y can not be a straight line, as was at first suspected, but a curve whose slope is an exponential function involving the length and the mean temperature of the two air columns. If the original function were linear, the first differential would, of course, be a constant. The conclusion must be that within the limits of variation of the several variables in these equations, the variation of dy/dx is perhaps small enough to be negligible. This may be investigated by means of reasonable substitutions in these equations.

There occurs, moreover, in these equations, the term $d\theta_z/d\theta_{m-n}$ which one may interpret as the change of the mean temperature of the air column of length Z_z relative to the mean temperature of the air column of length Z_{m-n}. Since temperature varies so irregularly with altitude and time, it is obvious that the value of $d\theta_z/d\theta_{m-n}$ must vary almost incessantly in a most irregular manner. If the value of Z_{m-n} is chosen to be 1,000 meters (3,281 feet) and θ_{m-n} the mean temperature of this stratum which lies between 1,000 and 2,000 meters above sea level (3,281 to 5,562 feet), it seems reasonable that, for stations in the central and eastern United States, this stratum would occupy a relatively intermediate position in the stratum between the surface and some high level such as 3, 4, or 5 kilometers (9,842, 13,123, and 16,404 feet) above sea level. One should expect that temperature changes within the stratum (m-n) would proceed at about the same rate as within a stratum extending from the surface to, say, 3 kilometers above sea level (13,123 feet), and that, on the average, they would be relatively more rapid as higher and higher levels are considered as upper boundaries to the thicker air stratum. If, therefore, one wishes to assume a value for the fraction $d\theta_z/d\theta_{m-n}$, it would appear that for the 3-kilometer level (9,482-foot) the value 0.90 is appropriate, for the 4-kilometer level (13,123-foot) the value 0.80, and for the 5-kilometer level (16,404-foot) the value 0.65. (Attention is invited to the fact that the arbitrary assumption of a numerical value of $d\theta_z/d\theta_{m-n}$, while perhaps not serving to compute strictly accurate values of dy/dx, will not operate to influence the form of the curve.) Making the appropriate assumptions as to temperature, table 1 contains the results of computation from equation (5a) of values of dy/dx for several lengths of air column.

TABLE 1.—*Values of dy/dx computed from equation (5a) for various temperatures and lengths of air column and assumed values of $d\theta_z/d\theta_{1-2}$.*

Assumed mean temperature of air column (°C.).				Length of air column (meters).		
θ_{1-2}	θ_3	θ_4	θ_5	3,000	4,000	5,000
−22	−23	−25	−27	2.069	2.164	1.937
4	2	0	−2	2.136	2.260	2.045
30	27	25	23	2.190	2.340	2.141

It is seen that the values of dy/dx are very close to 2.000 and that the variation is not great. This may be seen more clearly if one examines the actual variation of angle of slope of the line representing the relation between x and y. For any given level the slope increases with increase of temperature, hence the difference between the angles of slope corresponding to the extremes of temperature will indicate the degree of curvature likely to be encountered in the curve of the original function. These angles and their differences appear in Table 2.

TABLE 2.—*Slopes of tangents to curve of $y=f(x)$ at temperature extremes, and their differences.*

θ_{1-2} (°C.)	Slope of tangent.		
	3,000 meters.	4,000 meters.	5,000 meters.
	° ′	° ′	° ′
−22	64 12	65 12	62 42
30	65 28	66 51	64 58
Difference	1 16	1 39	2 16

It can be seen at once that the difference between a straight line and a portion of a curve the maximum curvature of which would produce an angle of the order of 2° between tangents at its extremes is negligible. Figure 3 shows the relation between dy/dx and θ_{1-2} for the three levels. On the left-hand scale are values of dy/dx and on the right-hand scale are angles of slope.[5]

The conclusion to be drawn from these considerations is that *within the limits of natural variation of the mean temperature of the air columns involved, the relation between P_z/P_s and P_m/P_n is practically linear, when these pressures occur within the lowest 5 kilometers of the atmosphere; and, therefore, that an equation of that form may be employed for the computation of one of the pressures when the remaining three are known.* For convenience, this may be referred to as the Law of Pressure Ratios.

OBSERVATIONAL DATA.

Nature of the data.—The data employed in this study were obtained from the original records on file in the Aerological Division of the Weather Bureau. They include observations made at each of the aerological stations which have been, or are being, operated by the Weather Bureau.

---

[5] The apparently nonuniform change of the values of dy/dx with change of Z_z is chiefly the result of the value of $d\Theta_z/d\Theta_{1-2}$ chosen for the several levels. If it were not for the assumed decrease of $d\Theta_z/d\Theta_{1-2}$ with increase of elevation, the several curves in figures 3 should lie one above another in the order of the level for which they are characteristic. But since this factor is assumed to decrease with elevation, the 5-km. curve and the 3-km. curve both lie below the 4-km. curve. Since the law of variation of $d\Theta_z/d\Theta_{1-2}$ with Z_z is not known, it is not possible at this stage of the paper to verify the assumed values of the factor.

In order to render the results of this paper comparable with those of previous investigations on kindred phases of this subject, all the observations selected were made at approximately 8 a. m., 75th meridian time. Certain corrections, devised to yield pressure values which probably would have been measured had it been possible to make simultaneous observations at all levels, were employed. However, cases in which the surface pressure was changing rapidly during the kite flight were not used owing to the unreliability of extrapolating the pressures

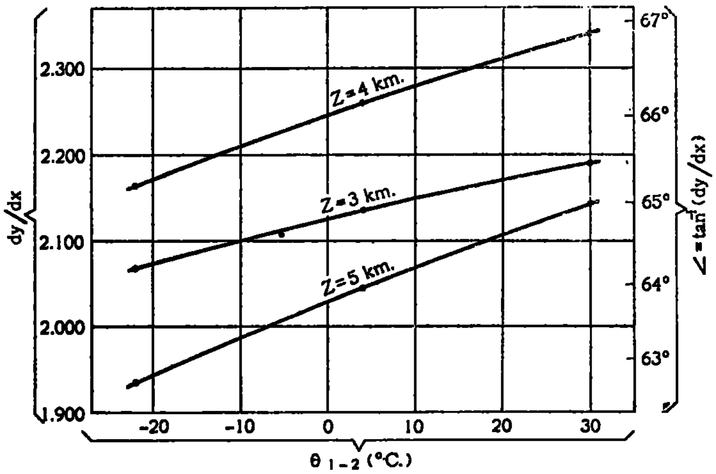

FIG. 3.—Relation between the mean temperature of the stratum of 1 kilometer thickness and the slope of the curve $y=f(x)$.

under such conditions. In other words, the pressure values from which ratios were formed are believed to be truly representative of simultaneous pressure conditions at all levels.

Having secured these data, ratios were formed between the pressures at 1 and 2 kilometers (3,281 and 6,562 feet) above sea level ($x=p_2/p_1$), and between pressures at the surface and some selected higher level ($y=p_z/p_s$). The higher levels thus selected were 3, 4, and 5 kilometers (9,842, 13,123, and 16,404 feet) above sea level, and, for each station, the investigation was carried as high as the amount of data justified.

Number of observations.—Table 3 gives the number of observations upon which the results of the portion of the paper employing current observations are based.

TABLE 3.—*Number of observations upon which results of current observations are based.*

Station.	With reference to level—		
	3 km. above m. s. l.	4 km. above m. s. l.	5 km. above m. s. l.
Ellendale, N. Dak.	72	79	....
Drexel, Nebr.	103	103	33
Broken Arrow, Okla.	72	....	....
Groesbeck, Tex.	90	62	....
Royal Center, Ind.	83	34	....
Mount Weather, Va.	91	91	38
Due West, S. C.	72	34	....
Leesburg, Ga.	45	....	....

These figures do not represent all available data in certain cases, especially with reference to the 3-kilometer level, but it is believed that they were sufficiently numerous to show accurately the relationships involved. This fact was obvious when the data were plotted in the form of dot charts. It is true, however, in the case of the 5-kilometer level that there were too few observations at most stations, and for that reason only Drexel and Mount Weather were used.

It is obviously unnecessary to reproduce all the dot charts. One is given in Figure 4, in order that the reader may see a typical example of the manner in which the observations distribute themselves. This type of distribution is characteristic of all stations and all levels.

Values of the constant a for aerological stations.—Having the data thus prepared and plotted, the line of good fit to the data—assuming, as was shown to be justified in the previous section, that a straight line adequately

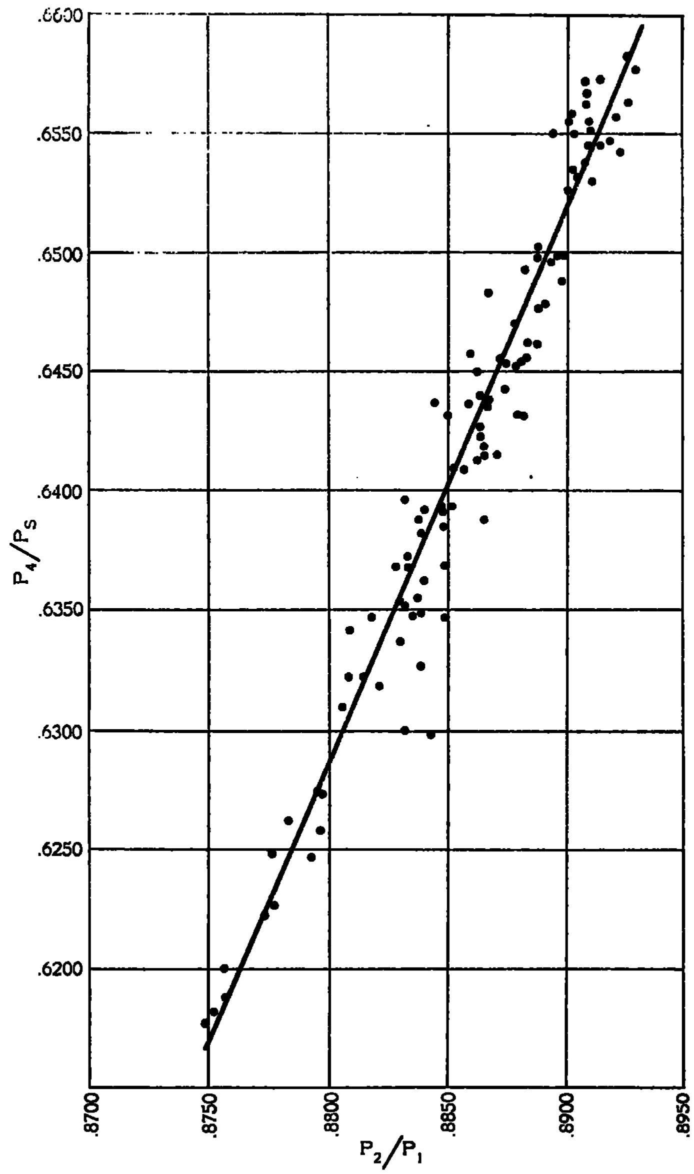

FIG. 4.—Distribution of individual observations about a line of good fit. Data from Royal Center, Ind.

represents the relationship between x and y—was determined by the ordinary method of least squares. This involves the evaluation of the two constants in the linear equation

$$y=ax+b,$$

in which x and y have the same significance as in the previous section, namely, $x=p_2/p_1$ and $y=p_z/p_s$.

The values of the constant a, where $a=dy/dx$, the slope of the line, were determined by the equation common in statistical procedure:

$$a = dy/dx = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

in which n is the number of observations. Table 4 contains the values of this constant.

TABLE 4.—*Values of the constant a, determined by the method of least squares.*

Station.	With reference to level—		
	3 km. above m. s. l.	4 km. above m. s. l.	5 km. above m. s. l.
Ellendale, N. Dak.	2.003	2.263	
Drexel, Nebr.	1.916	2.336	1.994
Broken Arrow, Okla.	1.976		
Groesbeck, Tex.	2.425	2.867	
Royal Center, Ind.	2.025	2.238	
Mount Weather, Va.	1.935	2.206	1.879
Due West, S. C.	1.928	2.113	
Leesburg, Ga.	2.710		

Values of the constant b for aerological stations.—The values of the second constant in the linear equation were determined by means of the following equation:

$$b = \frac{y - a(\Sigma x)}{n}$$

in which the values of a were those contained in Table 4. The following table contains the resulting values of b.

TABLE 5.—*Values of the constant b, determined by the method of least squares.*

Station.	With reference to level—		
	3 km. above m. s. l.	4 km. above m. s. l.	5 km. above m. s. l.
Ellendale, N. Dak.	−1.0423	−1.3594	
Drexel, Nebr.	−0.9696	−1.4275	−1.2016
Broken Arrow, Okla.	−1.0357		
Groesbeck, Tex.	−1.4414	−1.9161	
Royal Center, Ind.	−1.0804	−1.3532	
Mount Weather, Va.	−0.9744	−1.3024	−1.0911
Due West, S. C.	−0.9946	−1.2427	
Leesburg, Ga.	−1.6987		

Probable errors of single computations by the linear equation.—With the constants thus determined it is possible to form equations, characteristic of the various levels and several aerological stations, and to compute, using the original observed values of p_s, p_1, and p_2, values of p_z. In this way, values of the probable error of a single computation were derived, and these values are contained in Table 6. The equation for probable error was the one customarily used:

$$r = \pm .6745 \sqrt{\Sigma v^2/(n-1)}$$

in which v is the difference between a computed and an observed value of p_z.

TABLE 6.—*Values of the probable error of a single computation of pressure at stated free-air levels based upon empirical equations derived above. (Inches).*

Station.	With reference to level—		
	3 km. above m. s. l.	4 km. above m. s. l.	5 km. above m. s. l.
Ellendale, N. Dak.	±0.027	±0.050	
Drexel, Nebr.	±0.038	±0.047	±0.052
Broken Arrow, Okla.	±0.032		
Groesbeck, Tex.	±0.041	±0.042	
Royal Center, Ind.	±0.030	±0.041	
Mount Weather, Va.	±0.027	±0.038	±0.054
Due West, S. C.	±0.034	±0.039	
Leesburg, Ga.	±0.036		
Mean	±0.033	±0.043	±0.053

The probable error of a.—From equation (1) it follows, if p_s, p_2, and p_1, are regarded as free from error, that

$$dp_z = (p_s p_2 da/p_1) + p_s db \quad \text{.............} (6)$$

But since

$$b = \frac{(\Sigma y) - a(\Sigma x)}{n}$$

$$db = -(\Sigma x) da/n \quad \text{..................} (7)$$

whence,

$$dp_z = (p_s p_2 da/p_1) - p_s[(\Sigma x) da/n]$$

$$= [(p_s p_2/p_1) - p_s(\Sigma x)/n] da$$

and

$$da = dp_z/[(p_s p_2/p_1) - p_s(\Sigma x)/n] \quad \text{......} (8)$$

Now, one of the laws of the propagation of error states that [6] if $Z = Az$, in which A is a constant, then,

$$R = Ar$$

in which R is the probable error of Z and r is the probable error of z. It will be seen that this equation is of the same form as (8).

Since it is desired only to obtain an idea of the order of magnitude of the error in a, and since a is to be used without respect to season, it is possible to substitute for dp_z the average probable error for the several levels contained in the last line of table 6, mean annual values of p_s, p_1, and p_2 for a representative station, and representative values of $\Sigma x/n$ obtained from the computation sheets. In this way we may regard da as the probable error of a, and it proves to be, on the average,

$$\text{for the} \begin{cases} \text{3-km. level, } \pm 0.0012. \\ \text{4-km. level, } \pm 0.0016. \\ \text{5-km. level, } \pm 0.0020. \end{cases}$$

Thus, since nearly all the values of a are very close to 2.000, it is seen that the values as stated to three decimal places in Table 4 are justified.

[6] Merriman, Mansfield: *Method of least squares.* New York, 1915, p. 77.

The probable error of b.—Having the above values for a, it is now a simple matter to substitute in equation (7) and compute the probable error of b. These values prove to be:

$$\text{for the}\begin{cases}\text{3-km. level, } \pm 0.000008.\\ \text{4-km. level, } \pm 0.000010.\\ \text{5-km. level, } \pm 0.000013.\end{cases}$$

Conclusions concerning probable errors.—An interesting and highly pertinent fact becomes apparent upon consideration of the probable errors shown above: The large probable errors occur in the less significant member of this pair of constants. For example: When the angle of slope is close to 60°, as is the case here, a variation of one unit in the second decimal place of a implies a difference of slope of only about 7′. This is negligibly small. But a very slight variation in the value of b means that every ordinate will be in error by just that amount; or, to consider it geometrically, the entire line which represents the relation between x and y will be shifted parallel to itself by the amount of the error. These facts are important in attempting to carry over to the nonaerological stations the determination of these constants.

Another method of determining b.—The foregoing discussion has been built wholly upon the groups of current kite observations selected to represent the various levels and stations. The reliability of the values of the constants obtained by least-square analysis is dependent upon how closely the selected observations, taken in the aggregate, represent any observation that has been or will be made at a given station with respect to a given level. In other words, how closely does the mean of a given group of observations resemble the annual mean of the same elements based upon all the observations that are available, when the annual mean is also representative of conditions at 8 a. m., 75th meridian time?

It is evident that the graphical representation of the relation between x and y, should be a line passing through the annual mean corresponding to the same time of day.

It is stated by Mr. W. R. Gregg [7] that —

* * * The average time of the kite flights is such that the mean values of the meteorological elements at the surface are very nearly the same as the 24-hour averages. The differences are, in general, so small that it is deemed unnecessary to publish them in detail * * *.

Now, from Bigelow,[8] it is possible to obtain figures for correcting the 24-hour mean (annual) to the 8 a. m. mean (annual) and the corrected means are given in the following table. The corrections given by Bigelow are, of course, for the surface only, but since from (2),

$$dp_z = \exp \frac{-Z_z}{K\ (1+\alpha\vartheta_z)}\ dp_s$$

we can compute quite satisfactory values of the correction to be applied for the higher levels, we may reduce all the annual means for 24 hours to the annual means for 8 a. m.

[7] *Op. cit.*, p. 3.

[8] Bigelow, Frank H.: *Report on the barometry of the United States, Canada, and the West Indies.* Report of the Chief of the Weather Bureau, 1900-1901, Table 27, pp. 140-164.

TABLE 7.—*Annual mean pressures at the surface and several free-air levels. (8 a. m.) (mb.).*

Station.	Surface.	1 km.	2 km.	3 km.	4 km.	5 km.
Ellendale, N. Dak	963.3	899.5	795.2	701.4	617.3	541.7
Drexel, Nebr	969.5	901.0	797.7	704.7	621.4	546.9
Broken Arrow, Okla	989.7	903.4	801.5	709.4	627.0	551.6
Groesbeck, Tex	1,000.7	904.5	803.2	711.7	629.0	554.3
Royal Center, Ind	991.4	902.8	798.8	705.6	621.7	546.5
Mount Weather, Va	956.3	903.1	799.2	705.7	621.7	545.7
Due West, S. C	994.3	905.4	802.6	710.2	627.0	....
Leesburg, Ga	1,009.4	906.7	804.6	712.9	630.4	554.0

From these data it is possible to derive values of x, (p_2/p_1), and y, (p_z/p_s), and the values of these ratios are given in Table 8.

TABLE 8.—*Values of x and y for the annual mean at various aerological stations and for various levels.*

Station.	p_2/p_1	p_3/p_s	p_4/p_s	p_5/p_s
Ellendale, N. Dak	0.8840	0.7281	0.6408	0.5623
Drexel, Nebr	0.8853	0.7269	0.6409	0.5641
Broken Arrow, Okla	0.8872	0.7168	0.6335	0.5573
Groesbeck, Tex	0.8880	0.7112	0.6286	0.5539
Royal Center, Ind	0.8848	0.7117	0.6271	0.5512
Mount Weather, Va	0.8849	0.7379	0.6501	0.5706
Due West, S. C	0.8865	0.7143	0.6306	....
Leesburg, Ga	0.8874	0.7063	0.6245	0.5488

Now, from equation (1) we know that

$$b = y - ax = p_z/p_s - ap_2/p_1 \quad \text{............} \quad (1a)$$

from which it is possible, using values of a contained in Table 4 and of x and y contained in Table 8, to compute a new value of b. The graphical significance of this new value is that it is the y-intercept of a line having the same slope as that derived by least-square analysis, but passing through the point representative of the annual means contained in Table 7. Table 9 contains both the new value of b thus computed and the difference between this value and the corresponding value contained in Table 5.

TABLE 9.—*Values of b for the relation $y=f(x)$ characteristic of the annual mean.*

Station.	b_3	Diff.[1]	b_4	Diff.[1]	b_5	Diff.[1]
Ellendale	−1.0426	−0.0003	−1.3597	−0.0003	....	....
Drexel	−0.9693	.0003	−1.4272	.0003	−1.2012	0.0004
Broken Arrow	−1.0363	−.0006	....	....	....	....
Groesbeck	−1.4422	−.0008	−1.9173	−.0012	....	....
Royal Center	−1.0800	.0004	−1.3531	.0001	....	....
Mount Weather	−0.9744	.0000	−1.3020	.0004	−1.0921	−.0010
Due West	−0.9949	−.0003	−1.2426	.0001	....	....
Leesburg	−1.6986	.0001	....	....	....	....

[1] Value of b in Table 9 minus value in Table 5.

The differences are, in all cases, quite small; but since the values determined by each method have characteristic merits, it is thought that a mean of the two methods should give a value of greater reliability than either considered separately. Consequently, such a mean was formed in each case. The final values of b thus determined appear in Table 10. But before presenting them, there is the further consideration of the correct value of a to accompany the new values of b. It is clear that if the mean value of b differs from the value determined by the second method, a line passing through the mean b will not pass exactly through the point representing the annual mean of the ratios. In order to make it do this, it would be necessary to vary a slightly. This can be done by means of the equation

$$a = \frac{y-b}{x} = p_1[(p_s/p_s) - b]/p_2 \text{ . . . . } \quad (1b)$$

also derived from equation (1). Substitutions have been made in this equation, and the results, which may be regarded as the final values of a and b appear in Table 10.

TABLE 10.—*Final values of a and b for the various aerological stations.*

Station.	With respect to level—					
	3 km. above s. l.		4 km. above s. l.		5 km. above s. l.	
	a	*b*	*a*	*b*	*a*	*b*
Ellendale	2.003	−1.0424	2.263	−1.3595	....	....
Drexel	1.916	−0.9694	2.336	−1.4273	1.994	−1.2014
Broken Arrow	1.976	−1.0360	....	....	....	....
Groesbeck	2.424	−1.4418	2.866	−1.9167	....	....
Royal Center	2.025	−1.0802	2.238	−1.3531	....	....
Mount Weather	1.935	−0.9744	2.206	−1.3022	1.878	−1.0916
Due West	1.928	−0.9947	2.113	−1.2426	....	....
Leesburg	2.710	−1.6986	....	....	....	....

It is seen, when Tables 4 and 10 are compared, that there is very little difference between the values of a derived by the two methods. This indicates that the bodies of data which were treated by the method of least squares were truly representative of mean conditions.

Errors of computation attributable to errors of x.—Hitherto, in this paper, it was assumed that the values of x, i. e., p_2/p_1, were free from error. But, in practice, when use is made of pressures at these levels which are, themselves, the results of computation, it is clear that error must be introduced into the final result by the inaccuracies of the components of the value of x. The investigation of the accuracy of the method of obtaining p_1 and p_2 was referred to earlier [9] in this paper. From that investigation it is possible to obtain certain facts which will be useful here. For example:

(1) Errors in p_1 and p_2, if relatively large, were usually of the same algebraic sign.

(2) Errors of p_2 were usually about twice those of p_1.

(3) About 90 per cent of the errors of p_1 were less than 0.05 inch (1.69 mb.) and an equal percentage of errors of p_2 were less than 0.11 inch (3.73 mb.).

If p_s is considered free from error (it is probably the most precise of all meteorological measurements), and if a is regarded as subject to such small errors as are indicated by the probable errors discussed earlier (the error of b is probably negligibly small), we may derive from equation (1) the following expression from which the degree of error to be expected in p_x may be computed:

$$dp_x = p_s[(p_1 p_2 da + p_1 a dp_2 - p_2 a dp_1)/p_1^2] \text{ . . . . . . } \quad (9)$$

Let, for example, p_s be 950 mb.; p_1, 900 mb.; p_2, 800 mb.; and let a be 2.000; da, .001; dp_1, −1.5 mb.; dp_2, −3.5 mb.; and these, when substituted in the above equation give

$$dp_x = -3.7 \text{ mb. } (0.11 \text{ inch}).$$

[9] See footnote 2, p. 437.

If the above figures are again the same except for dp_1 and dp_2 which may be assumed to be small and of opposite sign, as for example, 0.5 mb., and −0.5 mb., respectively, we have,

$$dp_x = -1.1 \text{ mb. } (-0.03 \text{ inch}).$$

If the algebraic signs of these errors are reversed,

$$dp_x = 2.7 \text{ mb. } (0.08 \text{ inch}).$$

These results are sufficient to indicate the order of magnitude of the errors to be expected in computations of p_x. It is seen at once that, *regardless of the level to which the computations refer, the accuracy is just about as satisfactory at the high level as at the 2 km. level.* This is a fact which lends considerable importance to the law of pressure ratios as an effective means of computing pressures at higher levels.

Before undertaking the task of determining the constants of the linear equations for nonaerological stations, it will be of interest to compute a few pressures for these high levels at the aerological stations, using values of p_1 and p_2 that have been computed from surface conditions alone, and to compare the results with pressures measured by means of kites at those levels. This can be done very readily by referring to kite flights made during the period December 1, 1922, to February 28, 1923, during which time these free-air reductions were performed by a group of stations in the central and eastern United States in connection with a test of the efficacy of the method of computation. The Aerological Division has supplied the data, and Table 11 contains the comparisons and residual errors.

TABLE 11.—*Comparison for various aerological stations of pressures observed by means of kites, and pressures computed by the law of pressure ratios, employing computed pressures at 1 and 2 kilometers above sea level.*

Station.	Date.	Pressure at—							
		3-kilometer level.				4-kilometer level.			
		Observed (mb.).	Computed (mb.).	Difference.		Observed (mb.).	Computed (mb.).	Difference.	
				Mb.	In.			Mb.	In.
Ellendale, N. Dak.	1922. Dec. 1	693.4	691.8	+1.6	+0.05	608.0	605.6	+2.4	+0.07
	1923. Jan. 7	685.7	686.9	−1.2	− .03	601.6	602.2	−0.6	− .02
	Jan. 12	697.7	697.7	0.0	.00	613.4	611.5	+1.9	+ .06
	Jan. 17	694.3	696.0	−1.7	− .05	611.4	614.5	−3.1	− .09
	Feb. 24	695.0	698.2	−3.2	− .10	609.9	613.6	−3.7	− .11
	Feb. 25	691.0	691.0	0.0	.00	607.2	605.3	+1.9	+ .06
Drexel, Nebr.	1922. Dec. 22	698.7	698.1	+0.6	+ .02	616.1	615.2	+0.9	+ .03
	Dec. 25	697.1	698.7	−1.6	− .05	614.5	615.8	−1.3	− .04
	Dec. 28	699.6	702.1	−2.5	− .07	614.9	618.2	−3.3	− .10
	1923. Jan. 25	698.1	696.3	+1.8	+ .05	614.5	611.7	+2.8	+ .08
	Feb. 5	702.7	701.1	+1.6	+ .05	618.2	615.0	+3.2	+ .10
Broken Arrow, Okla.	1922. Dec. 19	708.7	709.5	−0.8	− .02	....	....	......	......
	Dec. 21	705.4	707.1	−1.7	− .05	....	....	......	......
	1923. Jan. 11	699.2	703.3	−4.1	− .12	....	....	......	......
	Jan. 19	710.7	713.5	−2.8	− .08	....	....	......	......
Groesbeck, Tex.	1922. Dec. 6	715.8	715.3	+0.5	+ .01	633.4	632.6	+0.8	+ .02
	Dec. 31	701.5	701.5	0.0	.00	617.6	617.5	+0.1	.00
	1923. Jan. 23	701.6	704.9	−3.3	− .10	616.0	622.4	−6.4	− .19
	Feb. 28	704.0	709.4	−5.4	− .16	619.0	625.4	−6.4	− .19
Royal Center, Ind.	1922. Dec. 24	699.1	699.9	−0.8	− .02	614.9	613.5	+1.4	+ .04
Due West, S. C.	1923. Jan. 22	709.0	712.1	−3.1	− .10	625.1	628.7	−3.6	− .11

This table is self-explanatory and calls for very little comment. The number of cases is so small that no attempt will be made to treat the residuals statistically. The object is rather to show the general magnitude of the departures of the computed pressures from the observed. This, it will be seen, is about the same as for the 2-kilometer level, even at the 4-kilometer level. The reader, if he is familiar with the discussion of the accuracy of the 1 and 2 kilometer maps, will recall that one of the difficulties of using aerological stations in combination with nonaerological stations for making the isobaric maps was that the surface temperature at the former was often too low relatively owing to the exposure of thermometers so near the ground. This had its reflection in pressures at 1 and 2 kilometers that were likewise low. In a like manner, pressures at these two levels, which are themselves too low, will produce similar results in the pressure at 3 or 4 kilometers. It is gratifying to note, however, that, even in cases where the error at 1 and 2 kilometers was rather large, the error at the high levels was not any greater than that at 2 kilometers. This indicates that if we accept the 2-kilometer map as correctly indicative of barometric conditions at that level, we must also accept the 4-kilometer chart, for it is possessed of no greater error.

THE DETERMINATION OF CONSTANTS FOR NONAEROLOGICAL STATIONS.

The constant a.—It was learned from equation (5) that the value of dy/dx, or a, is a function of the mean temperatures of the two air columns, one extending from the surface to the highest level under consideration, and the other from the 1 to the 2 kilometer level, when Z_s is regarded as a parameter. It is obvious that one means of approaching the problem of the determination of a for interlying nonaerological stations is to determine for each aerological station and each level the numerical value of the differential coefficient $d\theta_s/d\theta_{1-2}$. This value could then be reduced to correspond to air columns of uniform length at the various stations, which would eliminate the factor Z_s and leave the differential term subject to temperature alone, which is a variable having a marked geographic distribution. Thus the values of $d\theta_s/d\theta_{1-2}$ could be plotted geographically and interpolations made for the nonaerological stations. Thence by a process which is the reverse of that mentioned above for obtaining values corresponding to uniform lengths of air column, the value could be determined for the length of air column characteristic of each station. Finally, by the substitution of proper values for the temperatures, the value of dy/dx for nonaerological stations could be computed. Such a process, while it possesses the merit of permitting a to be *computed*, is indirect. There is a shorter method which is, however, akin to that sketched above.

This second plan is to deal directly with the values contained in Table 10. As was stated above, these values are dependent upon several factors—the mean temperatures of two air columns, and the lengths of two air columns. The temperature terms have a geographical distribution, but the length of the long air column Z_s is characteristic of each station. In order to make the values of a, empirically derived, as was shown above, comparable with one another, one must first eliminate the effect of the different characteristic values of Z_s at the various stations. This may be done graphically.

If one plots the value of a as an ordinate, and the value of Z_s as an abcissa, one obtains a curve showing the relation between these two factors. When such a curve is plotted for each aerological station, it is possible to take from the graph a value of a which would correspond to any desired length of air column at any station. Thus, for each station, have been taken from the curves in Figure 5, a series of values of a between 2,750 and 4,000 meters. This gives, for a given length of air column, values of a for each station corresponding to exactly the same length of air column. In other words, the values of a are those which would have come from the least square analysis had a uniform length of air column been used at each station instead of certain levels above sea level. In Figure 5, the values of a are given on the left-hand scale of ordinates; the corresponding angle of inclination of the straight line appears on the right-hand scale of ordinates. The scale at the bottom of the figure shows the length of air column in meters, and at the top in feet.

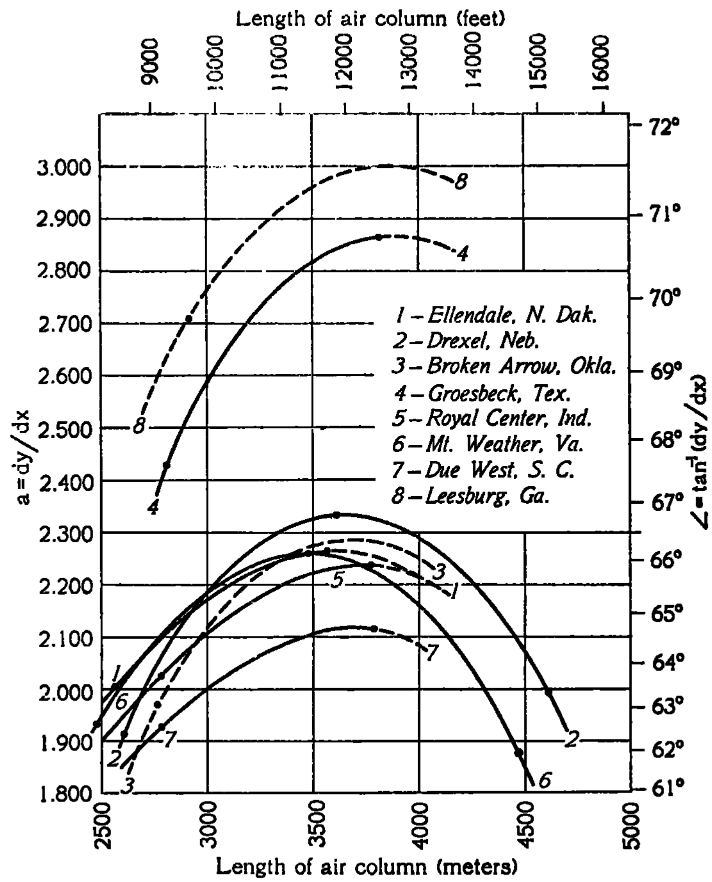

FIG. 5.—Relation between the slope of the line $y=f(x)$ and the length of air column for the several kite stations.

TABLE 12.—*Values of a obtained graphically for uniform lengths of air column at the various aerological stations.*

Station.	Length of air column (meters).			
	2,750	3,000	3,500	4,000
Ellendale, N. Dak	2.085	2.171	2.262	2.217
Drexel, Nebr	2.033	2.185	2.327	2.287
Broken Arrow, Okla	1.960	2.120	2.275	2.247
Groesbeck, Tex	2.358	2.581	2.813	2.861
Royal Center, Ind	2.015	2.106	2.220	2.214
Mount Weather, Va	2.086	2.184	2.260	2.160
Due West, S. C	1.915	2.001	2.105	2.084
Leesburg, Ga	2.583	2.760	2.961	2.995

These data, being subject only to geographical variation, may be plotted upon maps and lines of equal value

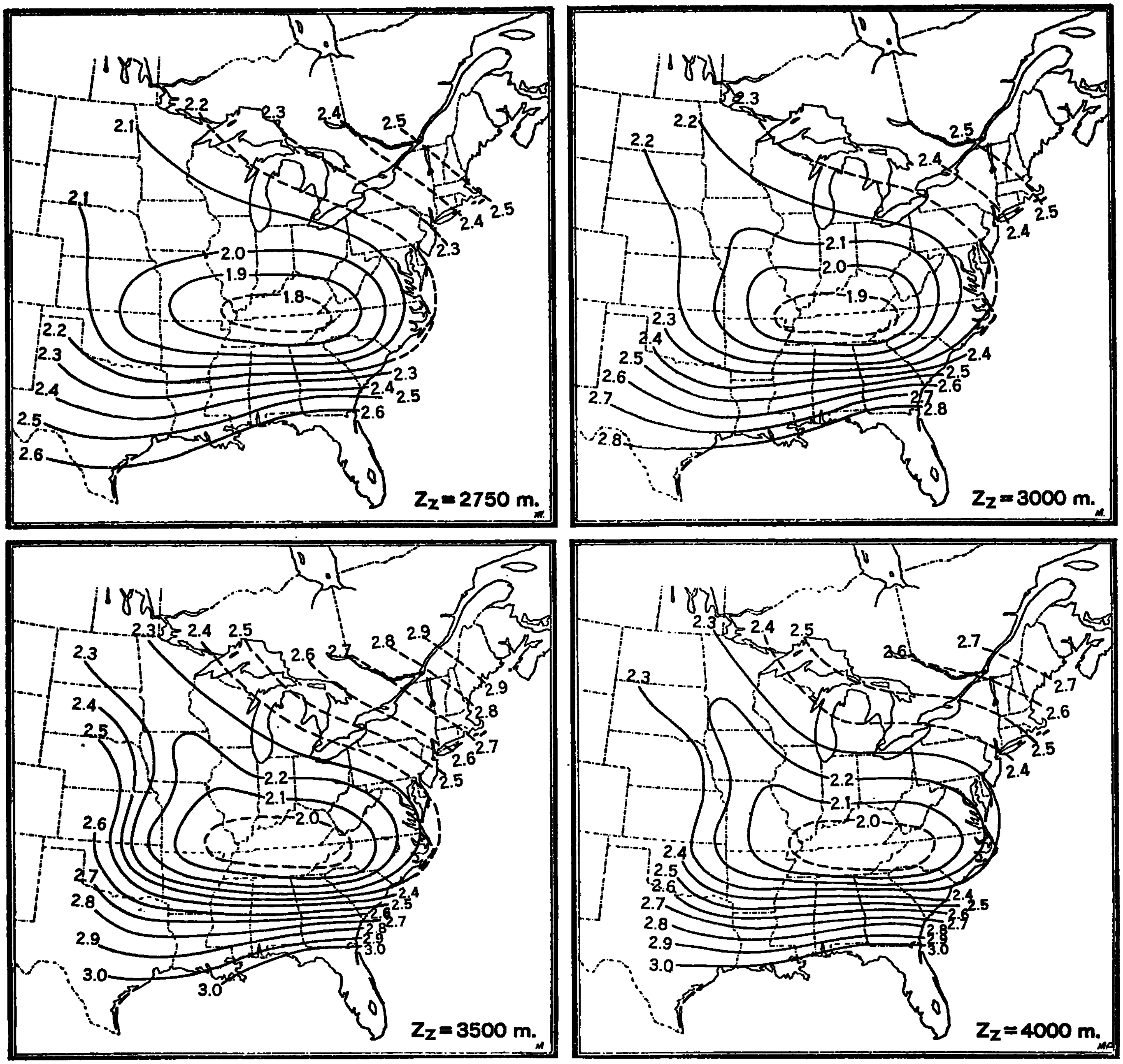

FIG. 6.—Geographical distribution of the slope of the line $y=f(x)$.

of a may be drawn. Figure 6 contains four maps, one for each column in Table 12. The length of air column to which it refers appears in the lower left-hand corner.

Peculiarities of the geographical distribution of a.—The astonishing apparition upon these charts of a region of low value of a through the middle latitudes of eastern United States, accompanied by a steep horizontal gradient to the south and a slight gradient to the north, is the just cause for serious meditation. One can consider the slope of the line $y=f(x)$ as a function either of mean temperatures of the two air columns or of four pressures occurring at the ends of these columns. But however one may look at it, there is difficulty in seeing why dy/dx should, as one progresses from north to south, first decrease slowly and finally increase rapidly. If we consider temperature, we have an element which has a pronounced geographical distribution, from low in the north to high in the south, not only at the surface but at free-air levels also. The effect of such a regular gradient would be to produce an increasing value of the slope with decrease of latitude in this region. Again, with reference to pressure, it is found that considering monthly mean pressures the intensity of the south-to-north horizontal gradient in the free air increases with elevation, while it is very slight at the surface. The gradients at 1 to 2 kilometers above sea level lie at intermediate positions between the surface and the high level under consideration. Consequently, when ratios are formed they also increase regularly from north to south and the result is the same as under the consideration of temperature.

We have reason to believe, as was shown earlier, that the values of a derived by least-square analysis are correct physically, and they have been submitted to careful arithmetical check to verify the computations.

Upon what may we place the responsibility for this surprising characteristic of the charts? In the equation for dy/dx attention was called to the term $d\vartheta_n/d\vartheta_{m-n}$, the relative change of the mean temperatures of the two air columns. This ratio occupies a highly strategic position in the equation, since all terms are multiplied by it. It is altogether possible that the evaluation of this term from the diurnal variation of temperature at the several stations would afford a complete and adequate explanation of the phenomenon. But, at this time, data on the diurnal variation of temperature have not been worked up into final form, and such explanations as might be given would, of necessity, be based upon assumptions as to the nature of the diurnal march at the various levels. It does not appear that such procedure would be justifiable.

The explanation of the horizontal distribution of the values of dy/dx must, therefore, await the time when data are available. The problem is one which must ultimately be discussed, yet which is not decidedly pertinent to the present paper.

Values for nonaerological stations.—Upon the basis of the four charts in Figure 6, one may tabulate the values of a corresponding to air columns of the several lengths indicated by the column heads in Table 12. Replotting these values, one may take from the new curves values of a corresponding to lengths of air column characteristic of the particular nonaerological station.

Since the data for the 5-kilometer level were not numerous, it has been thought best not to push too far the possibilities of map construction at levels above 4 kilometers. For this reason Table 13 contains only the values of a for the levels 3 and 4 kilometers above sea level for a selected list of stations at which it is desired to employ upper-air reductions.

TABLE 13.—*Values of a for various nonaerological stations.*

Station.	With reference to level— 3 km. above m. s. l.	With reference to level— 4 km. above m. s. l.	Station.	With reference to level— 3 km. above m. s. l.	With reference to level— 4 km. above m. s. l.
Burlington, Vt	2.535	2.684	Abilene, Tex	2.000	2.778
Boston, Mass	2.563	2.581	Oklahoma City, Okla	1.945	2.517
New York, N. Y	2.351	2.462	Omaha, Nebr	1.957	2.321
Pittsburgh, Pa	2.081	2.260	Little Rock, Ark	2.091	2.337
Washington, D. C	2.190	2.199	St. Louis, Mo	1.919	2.051
Norfolk, Va	2.260	2.213	Moorhead, Minn	2.076	2.247
Wilmington, N. C	2.291	2.294	Duluth, Minn	2.128	2.349
Charleston, S. C	2.534	2.628	Madison, Wis	2.090	2.276
Pensacola, Fla	2.789	3.008	Lansing, Mich	2.145	2.370
Birmingham, Ala	2.201	2.472	Indianapolis, Ind	1.905	2.110
New Orleans, La	2.779	3.008	Nashville, Tenn	1.795	1.949
Houston, Tex	2.674	2.981	Lexington, Ky	1.842	2.005
Palestine, Tex	2.438	2.840	Columbus, Ohio	1.961	2.185

The constant b.—While it is true that the value of the slope a is of great importance in the determination of free-air pressures, it has been shown that, for slopes of the magnitude found in this study, the tangent changes at such a rate that the variation of angle from one station to another is really but slightly significant. The constant b, however, is highly important, since it determines the y intercept. Its variation produces significant vertical displacements of the line $y=f(x)$.

It was shown that by the use of equation (1a) and mean annual pressure values very accurate values of b could be computed. Comparison of values for the kite stations was given in Table 9. It is likewise possible to substitute mean annual pressure values characteristic of the nonaerological stations in equation (1a) and values of a from Table 13, and thus derive a series of values of b.

The surface pressures used in Table 14 are taken from Bigelow's "Report on the barometry of the United States, Canada, and the West Indies" (see footnote 8), except in a few cases of stations at which the altitude of the barometer has been changed since 1900. The free-air pressures have been interpolated from graphs in Supplement 20 of the MONTHLY WEATHER REVIEW (see footnote 4).

TABLE 14.—*Values of b for various nonaerological stations.*

Station.	With reference to level— 3 km. above m. s. l.	With reference to level— 4 km. above m. s. l.	Station.	With reference to level— 3 km. above m. s. l.	With reference to level— 4 km. above m. s. l.
Burlington, Vt	−1.5436	−1.7591	Abilene, Tex	−1.0301	−1.8071
Boston, Mass	−1.5711	−1.6706	Oklahoma City, Okla	−0.9967	−1.5885
New York, N. Y	−1.3803	−1.5621	Omaha, Nebr	−1.0111	−1.4190
Pittsburgh, Pa	−1.1277	−1.3711	Little Rock, Ark	−1.1477	−1.4482
Washington, D. C	−1.2428	−1.3334	St. Louis, Mo	−0.9911	−1.1905
Norfolk, Va	−1.3050	−1.3455	Moorhead, Minn	−1.1202	−1.3572
Wilmington, N. C	−1.3323	−1.4170	Duluth, Minn	−1.1608	−1.4346
Charleston, S. C	−1.5476	−1.7119	Madison, Wis	−1.1326	−1.3819
Pensacola, Fla	−1.7744	−2.0501	Lansing, Mich	−1.1836	−1.4675
Birmingham, Ala	−1.2372	−1.5610	Indianapolis, Ind	−0.9718	−1.2383
New Orleans, La	−1.7658	−2.0508	Nashville, Tenn	−0.8810	−1.1006
Houston, Tex	−1.6702	−2.0245	Lexington, Ky	−0.9127	−1.1420
Palestine, Tex	−1.4520	−1.8914	Columbus, Ohio	−1.0226	−1.3058

Now, by means of Tables 13 and 14, it is possible to form an equation characteristic of each station for each of the two reduction levels. By means of these equations, the barometric pressure may be computed at each of these levels. For example, the two equations for Washington, D. C., are:

For the 3-km. level .... $p_3=p_s\left[2.190\left(\frac{p_2}{p_1}\right)-1.2428\right]$

For the 4-km. level .... $p_4=p_s\left[2.199\left(\frac{p_2}{p_1}\right)-1.3334\right]$

On February 7, 1923, at 8 a. m., 75th meridian time, the values of p_s, p_1, and p_2 were, respectively, 30.21 inches, 26.67 inches, and 23.47 inches. Substituting in the above equations the pressures at 3 and 4 km. above sea level are, respectively, 20.68 inches and 18.18 inches.

Constants for plateau stations.—The method of calculating the values of the constants a and b for stations in the Plateau region of western United States affords an interesting problem. This will be discussed in a later paper in which there will also be presented a method for obtaining pressures at 1 and 2 kilometers in that region. The combination of the several methods will enable one to prepare a free-air pressure chart for the entire United States with the exception of the low-lying coastal regions in the extreme West.

APPLICATION TO MAP DRAWING.

Comparison with observed winds.—Having the constants for the various nonaerological stations, it is only a matter of mechanical computing (which, in practice, can, of course, be greatly facilitated by the use of tables) to obtain pressures at 3 and 4 kilometers above sea level and draw isobaric charts. To facilitate the preparation of a few test charts, the Aerological Division has supplied free-air wind data obtained by means of pilot balloons for

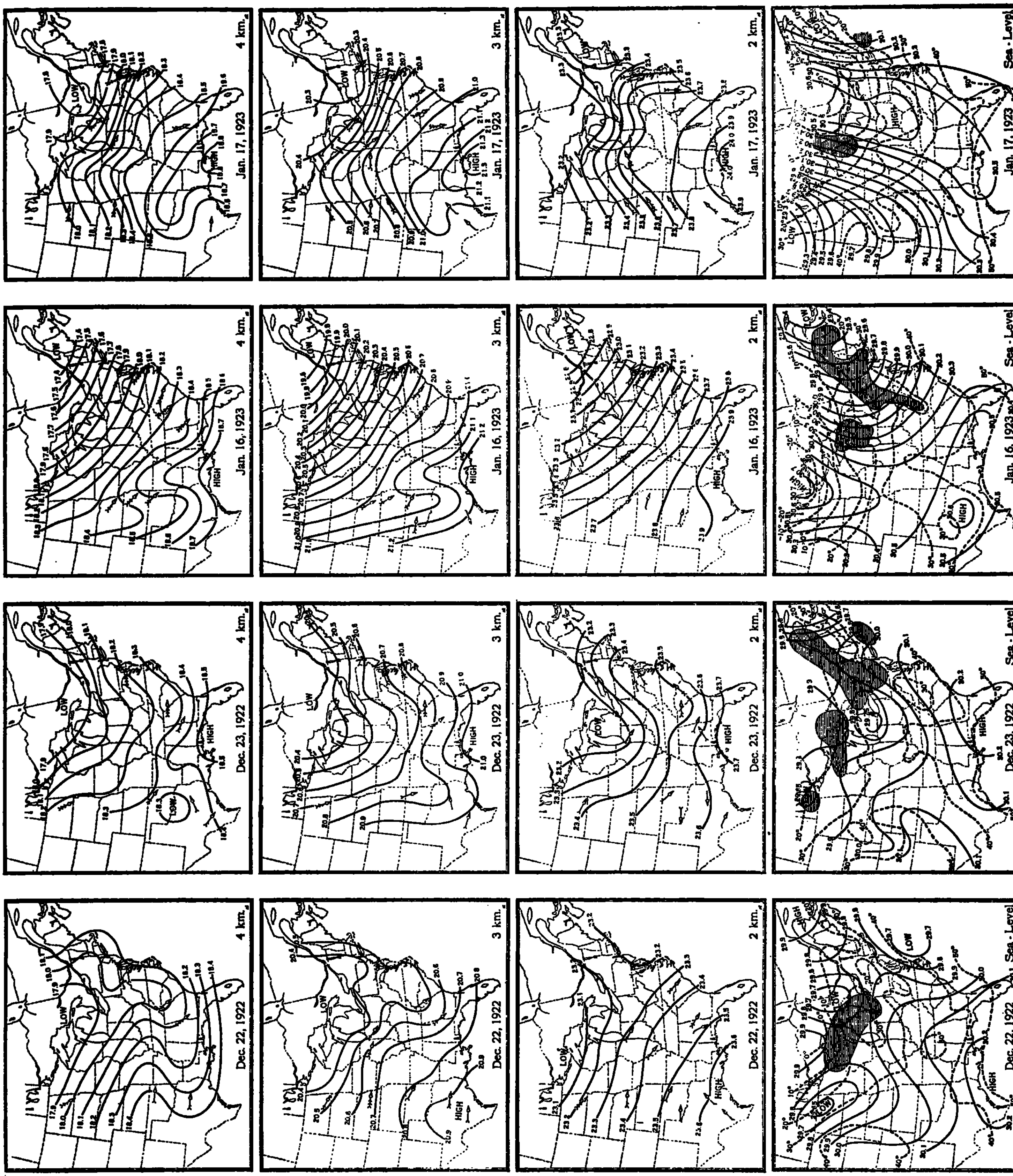

FIG. 7.—Weather maps for sea level and pressure maps at 2, 3, and 4 kilometers above sea level for December 22–23, 1922, and January 16–17, 1923. Wind arrows on free-air maps indicate the winds observed at aerological stations, each barb indicating a velocity of 10 miles per hour. Shaded areas on sea level charts indicate precipitation occurring during the 24 hours *preceding* time of the map.

December 22–23, 1922, and January 16–17, 1923, all of which dates lie within the period when the method for obtaining pressures at 1 and 2 kilometers was being tested by daily post card reporting.

The reduced pressures for these high levels in general distribute themselves smoothly, so that they are comparable in this respect with the charts for 1 and 2 kilometers. It is not doubted that the accumulation of data will render future revision of the constants desirable, but this is a matter which does not affect the validity of the method. Granting that slightly more smoothing is required for these charts than is practiced upon the sea level map, there is no mistaking the general trend of horizontal pressure gradients which are the significant and important factors in the determination of the movement of air masses. Isobaric irregularities at these high levels, are, it is believed, less significant than irregularities of the same magnitude on the sea level map.

It has been pointed out before that tests of these maps are very difficult, for one is never sure whether the records of pilot balloon flights are truly representative of air movement at the levels in question. This is not because of inherent inaccuracies in the methods of observing and reduction, but more probably because of the fickleness of wind conditions combined with the fact that the balloon record represents only a momentary observation at any particular level. We know that if the air is in adjustment with the pressure gradient in the free air, it will move nearly parallel to the isobars and with a velocity inversely proportional to the distance between isobars. When the wind direction is not parallel to the isobars, it does not necessarily condemn the isobars, for it may be that the air is not in perfect adjustment to the gradient, or that there are local deviations in the gradient which are so small as to escape record on the map. Local convective influences may greatly disturb the wind direction, yet be extremely local. Therefore, a lack of perfect agreement is not completely attributable to faulty isobars.

On the other hand, there is no desire to utilize this fact as an alibi. The investigator is as curious as the reader to know the true nature of the cause of such discrepancies as may exist; but it is doubtful whether one will ever be able definitely to establish an unimpeachable standard of isobaric accuracy in the free air, owing to the difficulty of simultaneous accessibility of a sufficiently large number of points. One must, in all fairness, judge the results of this map drawing in a broad way and attempt to compare large movements of air as indicated by simultaneous free-air wind directions at aerological stations with general isobaric trend. It is asking too much at this, and probably at any, stage of free-air map drawing to expect every wind direction to agree with the isobaric trend in its immediate vicinity. This does not always hold even at the surface close to the reduction level, although in that case we have recourse to turbulence, friction, and topography to explain the discrepancy; and such explanations can not be drawn upon in such large measure in the free air.

With these introductory remarks, Figure 7, showing the pressures at sea level, 2, 3, and 4 kilometers for December 22–23, 1923, and January 16–17, 1922, respectively, may be studied. On these charts are plotted the wind directions observed by means of pilot balloons at or near 8 a. m., 75th meridian time. These dates were selected because of the large number of ascents reaching the required altitudes. The speed of the wind in miles per hour is indicated by the barbs on the arrows, one barb indicating 10 miles of wind per hour.

These charts are offered in the hope that the reader will study them and draw his own conclusions as to the agreement between wind direction and isobaric trend, and the relation between the barometric configurations at the several levels. It is recognized that the number of maps is small, but available time and space hardly justify the presentation of more. No attempt is made to discuss the physical relations between wind conditions at the several levels, since the object of this paper is to treat only of the method of reduction and its application to map drawing. It will be conceded, it is hoped, that the Law of Pressure Ratios is one of theoretical interest and considerable practical importance. It has the merit of being founded on firm theoretical grounds and confirmed by observation so closely that the final error of computation is largely a function of the pressure errors at 1 and 2 kilometers above sea level—and these have been shown to be satisfactorily small.

ACKNOWLEDGMENTS.

The author wishes to thank Mr. E. W. Woolard, for his courtesy and helpfulness in discussing and verifying the equations employed, and members of the Aerological Division of the Weather Bureau for their very willing and enthusiastic assistance at every point where collaboration was requested.

THE WINDS OF OKLAHOMA AND EAST TEXAS.

By JOHN A. RILEY, Meteorologist.

[Aerological Station, Broken Arrow, Okla., September 26, 1923.]

SYNOPSIS.

Some of the outstanding features of surface and free-air winds over Oklahoma and east Teaxs are presented in tables and graphs. The data are mainly based on four years' pilot balloon records at three stations: Broken Arrow and Fort Sill, Okla., and Groesbeck, Tex., with a total of 7,075 flights. The paper does not aim at completeness for all phases of the wind even for the region covered; an exhaustive compilation of the data for this and other geographic groups is to be published later by the Aerological Division as Part II of *An Aerological Survey of the United States.*

Notable features of the winds of this group are: At the surface, largely predominating south winds in summer and alternate north and south winds in winter, with a small percentage of east and west winds in all seasons. In the free air, a clockwise shift, with one exception, into a pronounced westerly drift aloft in all seasons; a north component amounting to more than 50 per cent at 4,000 meters and higher over the whole region in all seasons. The one exception is the summer winds of Texas in which a counterclockwise shift occurs, the wind having a northeasterly drift above 4,000 meters.

Graphs have been drawn to show the mean seasonal direction and velocity at the three stations; the percentage frequency of directions for summer, winter, and the year at four selected levels; the annual march of wind speeds based on monthly averages for the region as a whole; features of the diurnal march and the nocturnal stratification of speeds at low altitudes; and the frequency of high winds at ordinary flying levels.

Free-air winds are best studied in geographic groups such that conditions are nearly uniform throughout the group but differ in some particulars from conditions in other groups. Upper air conditions at various levels are also more uniformly distributed than are those at the surface, so that the network of aerological stations need